MATCHING AN EXPLOSIVE-MAGNETIC GENERATOR TO A RESISTIVE LOAD

USING A TRANSFORMER

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INTRODUCTION

The use of explosive-magnetic generators in modern technology and scientific investigations as pulsed generators of high-power electrical energy has been discussed in [1-3]. The circuit for directly connecting the explosive generators into a resistive load is the simplest and most convenient, but it has limitations both with respect to the value of the load resistance [4-6] and with respect to the value of the voltage developed [7]. In this connection, in [3] a current breaker is regarded as a necessary element in the circuit of an explosive electrical generator. We show below that the use of a pulsed matched transformer enables one to reduce these limitations considerably and to obtain satisfactory matching in many cases between the explosive-magnetic generator and a resistive load without a current breaker.

1. The Electric Circuit and the Equations of the Device

Using the electrical model of an explosive-magnetic generator as in [1, 4, 5], the circuit of the device is shown in Fig. 1. Here we have ignored all parasitic inductances connected in the inductance of the lead L₁. On the secondary side we have ignored the parasitic inductances, while the parasitic resistances are included in the load resistance r. The circuit is described by the equations

$$(L_{g}I_{1})' + (L_{l} + L_{1})I_{1}' + MI_{2}' = 0;$$
(1.1)

$$L_2I_2 + rI_2 + MI_1 = 0, (1.2)$$

where $M = k \sqrt{L_1 L_2}$ is the mutual inductance of the windings of the matching transformer, k is their coupling coefficient, and the primes denote differentiationwith respect to time. The energy given to the load while the generator is operating is

$$W_{\mathbf{L}\,1}=r\int_{0}^{T}I_{2}^{2}(t)\,dt,$$

where $T = l_g/D$ is the duration of the cycle, l_g is the length of the generator, and D is the velocity of the detonation wave. The mechanical work of the products of the explosion opposite to the electromagnetic forces

$$W_{\text{mech}} = -\frac{1}{2} \int_{0}^{T} I_{1}^{2}(t) \, dL \,. \tag{1.3}$$

The energy of the magnetic field of the system when T = 0 (at the end of the pumping)

$$W_{\rm M1} = \frac{1}{2} \left(L_{\rm g0} + L_{\rm I} + L_{\rm 1} \right) I_{\rm 10}^2 + \frac{1}{2} L_2 I_{\rm 20}^2 + M I_{\rm 10} I_{\rm 20}, \tag{1.4}$$

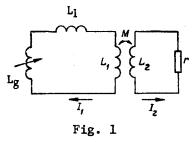
where $L_{go} = L_g(0)$; $I_{1o} = I_1(0)$; $I_{2o} = I_2(0)$. The energy of the magnetic field when t = T

$$W_{M2} = \frac{1}{2} \left(L_l + L_1 \right) I_{1m}^2 + \frac{1}{2} L_2 I_{2m}^2 + M I_{1m} I_{2m},$$

where $I_{1m} = I_1(T)$ and $I_{2m} = I_2(T)$ are the maximum values of the currents I_1 and I_2 . The energy balance equation for the working stage is

$$W_{\mathtt{wi}} + W_{\mathtt{mech}} = W_{\mathtt{w2}} + W_{\mathtt{L}1}.$$

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During the operation of the generator, part of the energy is stored in the windings of the matching transformer in the form of the energy of the magnetic field. After the completion of the working cycle, discharge of the transformer store of energy into the load begins. The discharge current is found by putting $L_g = 0$ in (1.1) and (1.2):

$$I_2 = I_{2m} \mathrm{e}^{-t/\tau_{\mathrm{d}}},$$

where $\tau_d = \tau (1 - [k^2/(1 + L_{l}/L_1)])$ is the discharge time constant, and $\tau = L_2/r$. From (1.1) we obtain

$$(L_l + L_1)I_1 + MI_2 = \text{const} = (L_l + L_1)I_{1m} + MI_{2m}$$

Hence it can be seen that not all the electromagnetic energy in the primary circuit is transferred to the load: as $t \to \infty$, $I_2 \to 0$ and $I_1 \to I_{1_{\infty}} = I_{1_{m}} + M/(L_{L} + L_1) \times I_{2_{m}}$. The energy transferred to the load in the discharge stage is

$$W_{\rm L} = r \int_{0}^{\infty} I_2^2(t) \, dt = \frac{1}{2} \, r I_{2m}^2 \tau_{\rm d} \, dt$$

The total energy in the load $W_L = W_{L1} + W_{L2}$. The electrical efficiency of the circuit is $W_L/(W_{M1} + W_{mech})$.

2. Optimization

It is well known [5] that the condition for maximum efficiency in transforming the energy of the explosion into electromagnetic energy is that the line current density in the busbar contact line should be constant. We will assume this condition is satisfied. Then $j = I_1/y = j_0$, where y(x) is the width of the busbar at a distance x from the origin. The optimization problem reduces to finding y(x) from Eqs. (1.1) and (1.2), taking into account the condition $I_1/y = j_0$ and the equation $L'_g = -\alpha/y$, where $\alpha = 2\mu_0 \delta D$, μ_0 is the magnetic permeability of free space, and δ is the gap between the busbar and the assembly. It can be shown that the solution of this problem is

$$y = y_0 e^{t/\tau}, \quad L_g = \frac{a\tau}{y_0} \left(e^{-t/\tau} - e^{-T/\tau} \right);$$
 (2.1)

$$I_1 = I_{10} e^{t/\tau}, \ I_2 = I_{20} e^{t/\tau}; \tag{2.2}$$

$$2\left(1+L_{l}/L_{1}-\frac{a\tau}{L_{1}y_{0}}e^{-T/\tau}\right)=k^{2};$$
(2.3)

$$I_{20} = -\frac{k}{2} \sqrt{\frac{L_1}{L_2}} I_{10}.$$
 (2.4)

Equation (2.3) follows directly from the energy balance equation for an explosive-magnetic generator with an exponential profile, while condition (2.4) must be ensured when pumping. From (1.3), (1.4), (2.1), (2.2), and (2.4) we obtain

$$W_{\text{mech}} = \frac{a\tau}{2y_0} \left(e^{T/\tau} - 1 \right) I_{10}^2;$$
(2.5)

$$W_{\rm M1} = \frac{1}{2} \left[L_{\rm g0} + L_l + \left(1 - \frac{3}{4}k^2\right) L_1 \right] I_{10}^2, \tag{2.6}$$

$$W_{\text{mech}} / W_{\text{M1}} = e^{T/\tau} \left[1 + \frac{L_l}{L_{g_0}} + \left(1 - \frac{3}{4} k^2 \right) \frac{L_1}{L_{g_0}} \right]^{-1}$$

A rational mode of operation corresponds to $W_{mech}/W_{Ml} >> 1$, whence it follows that $e^{T/\tau} >> 1$, $L_{\chi} \leq L_{go}$, $L_{1} \leq L_{go}$. The electrical efficiency is

$$\frac{W_{L}}{W_{M1} + W_{mech}} = k^{2} \frac{2 - \frac{k^{2}}{1 + L_{l}/L_{1}} - e^{-2T/\tau}}{4\left(1 + \frac{L_{l}}{L_{1}}\right) - 2k^{2}\left(1 + \frac{1}{2}e^{-2T/\tau}\right)} \cong \frac{k^{2}}{2\left(1 + L_{l}/L_{1}\right)}$$
(2.7)

and in the best case $(k = 1, L_{I}/L_{1} << 1)$ is 0.5; when k = 0,9 and $L_{I}/L_{1} << 1$, we obtain 0.4, and when k = 1 and L_{I}/L_{1} we obtain 0.25. Hence, the condition that the line current density should be constant enables one to transfer to the load not more than half of all the electromagnetic energy of the device. We will now formulate the optimization problem in the following form. We will assume that we are given r, the amplitude of the voltage pulse u_m , the pulse energy W_L , the energy of the pumping battery W_b , and the parameters δ , L_I , and k. We require to find L_1 , L_2 , y_0 , and $T = l_g/D$. It is more convenient, first of all, to introduce together with W_L the total energy of the system $W_t = W_{M1} + W_{mech}$. Although an accurate value of W_t is not known before solving the optimization problem, it can be obtained approximately from (2.7) by putting $L_I/L_1 = 0$: $W_L/W_t \cong k^2/2$. After the first solution we can refine W_t and again solve the problem. Thus, we can obtain four equations for the four required quantities. The first equation is (2.5), where $W_{mech} = W_t - W_b$. Equating the energy of the pumping battery W_b to W_{M1} , we then obtain the second equation, namely, (2.6), in which $W_{M1} = W_b$. The third equation is (2.3). For $u_m = rL_{2m}$, taking (2.4) into account, we can write the fourth equation:

$$\frac{u_m}{a_{j_0}} = \frac{k}{2} \frac{y_0 r/a}{\sqrt{L_2/L_1}} e^{T/\tau}.$$
(2.8)

We will solve the system of equations (2.5), (2.6), (2.3), and (2.8) for $L_{l} = 0$. Omitting the calculations, we will merely write the solution in the form

$$e^{T/\tau} = \frac{1}{2} \frac{W_{t}}{W_{b}} \left[1 + \sqrt{1 - \frac{k^{2}}{1 - k^{2}/2} \frac{W_{t}/W_{b} - 1}{(W_{t}/W_{b})^{2}}} \right];$$
(2.9)

$$L_{1} = \frac{k^{4}/8}{(1-k^{2}/2)^{3}} \frac{r^{2} (W_{t} - W_{b})/(aj_{0})^{2}}{(1-e^{-T/\tau})(u_{m}/aj_{0})^{4}};$$
(2.10)

$$\sqrt{L_2/L_1} = \frac{1 - k^2/2}{k/2} \frac{u_m}{aj_0};$$
(2.11)

$$\frac{y_{\theta}r}{u} = 4 \frac{1 - k^2/2}{k^2} \left(\frac{u_m}{aj_{\theta}}\right)^2 e^{-T/\tau}.$$
(2.12)

Taking into account expressions (2.11) and (2.12), we obtain

$$\frac{L_{g_0}}{L_1} = \left(1 - \frac{1}{2} k^2\right) \left(e^{T/\tau} - 1\right).$$

3. Discussion of the Results

It is seen from (2.7) that the efficiency is halved when k is reduced from 1 to 0.7 and when $L_{\tilde{l}}/L_1$ is increased from 0 to 1. The transformation factor $n = w_2/w_1 = \sqrt{L_2/L_1}$ (w₁ and w₂ are the number of turns in the primary and secondary windings of the matching transformer) is determined by the voltage and is independent of the pulse energy [Eq. (2.11)]. Equations (2.9)-(2.12) ensure assigned values of the energy W_L and the power u_m^2/r simultaneously. We will now consider the problem of the limiting power in the following formulation. We will assume W_t, W_b, k, and L_l to be fixed. Then as u_m is increased L₁ will fall as u_m^{-4} , and, according to (2.7), W_L will fall rapidly. Hence, it is seen that if it is necessary to transfer a certain minimum energy W_{Lmin} to the load, the limiting power corresponding to this energy is determined by the parasitic inductance of the lead L_l and by the coupling factor k [Eqs. (2.7) and (2.10)]. The dependence of L_l/L₁ on the peak power $p_m = u_m^2/r$ can be written in the form

$$\frac{W_{\rm t}}{W_{\rm b}}\frac{L_l}{L_1} = \frac{(1-k^2/2)^3}{k^4/8} \frac{p_m^2}{(aj_0)^2 W_{\rm b}/L_l}.$$

Suppose, for example, that $W_t/W_b = 20$ and k = 1. Then when $p_m = \alpha j_0 \sqrt{W_b/L_2}$, $L_2/L_1 = 0.05$, $W_L/W_t \approx 0.5$ and $W_L/W_b = 10$. If $p_m = 10\alpha j_0 \sqrt{W_b/L_2}$, then $L_2/L_1 = 5$, $W_L/W_t = 1/12$, and $W_L/W_b = 1.67$.

Finally, when $p_m = \alpha j_0 \sqrt{W_b/L_l}$ and k = 0.7 (instead of k = 1), we obtain $L_l/L_1 \approx 0.7$, $W_L/W_t \approx 0.15$, and $W_L/W_b = 3$ (instead of 10). In conclusion, we will give an example of the design of a generator and a matching transformer for a system with the following parameters: $r = 10 \ \Omega$, $u_m = 10^6 \ V$, $W_L = 10^6 \ J$, $W_b = 5 \cdot 10^4 \ J$, $L_l = 2 \cdot 10^{-9} \ H$, k = 0.8, $\delta = 4 \cdot 10^{-2} \ m$, $j_o = 4 \cdot 10^7 \ A/m$, and $D = 0.75 \cdot 10^4 \ m/sec$. We obtain $\alpha = 0.75 \cdot 10^{-3} \ \Omega \cdot m$, $\alpha j_0 = 3 \cdot 10^4 \ V$, $u_m/(\alpha j_0) = 33.3$, $W_L/W_t \approx k^2/2 = 0.32$, $W_t \approx 3 \cdot 10^6 \ J$, $W_t/W_b = 60$, $L_1 = 4.4 \cdot 10^{-8} \ H$, $L_l/L_1 \approx 0.05$, $T/\tau = 1n \ 60 = 4.1 \ \sqrt{L_2/L_1} = 56.6$, $L_2 = 1.41 \cdot 10^{-4} \ H$, $\tau = 1.41 \cdot 10^{-5} \ sec$, $T = 57.7 \cdot 10^{-6} \ sec$, $l_g = 0.43 \ m$, $y_0 r/\alpha = 0.74$, $y_0 = 5.5 \cdot 10^{-3} \ m$, and $y_m = 0.33 \ m$.

Hence, an explosive-magnetic generator has an exponential profile with initial ordinate $y_0 = 5 \cdot 10^{-3}$ m, a final ordinate $y_m = 0.33$ m, and a length $l_g = 0.43$ m. The matching transformer has a transformation factor $n \approx 57$, which corresponds to $w_1 = 1$, and $w_2 = 57$. The inductance of the primary winding $L_1 = 4.4 \cdot 10^{-8}$ H. A pumping battery with an energy of 5 $\cdot 10^4$ J should give a current $I_{10} = 2.2 \cdot 10^5$ A.

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